

1. (32 pts) Use mathematical induction to prove that:

a)
$$\frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{2n} = \left(1 - \frac{1}{2}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \dots + \frac{1}{2n-1} - \frac{1}{2n}$$

where $n \in P$

b) $8^{n+2} + 9^{2n+1}$ is divisible by 73 where $n \in N$

c) $5^n - 4n - 1 = 16k$ where k is an integer and n is a positive integer.

d) $1^2 - 2^2 + 3^2 - \dots + (-1)^{n-1} n^2 = \frac{(-1)^{n-1} n(n+1)}{2}$

2. (10 pts) Recursively define $a_0 = 1, a_1 = 3, a_2 = 5$ and $a_n = 3a_{n-2} + 2a_{n-3}$ for $n \geq 3$

Show $a_n = 2a_{n-1} + (-1)^{n-1}$ for $n \geq 1$

3. (10 pts) The Fibonacci numbers f_0, f_1, f_2, \dots are defined by $f_0 = 0, f_1 = 1$ and $f_n = f_{n-1} + f_{n-2}$ for $n = 2, 3, 4, \dots$

The lucas numbers l_0, l_1, \dots are defined by $l_0 = 2, l_1 = 1, \dots$ and $l_n = l_{n-1} + l_{n-2}$ for $n = 2, 3, 4, \dots$

Show that $f_n + f_{n+2} = l_{n+1}$ whenever n is a positive integer.

4. (6 pts) Establish the following logical equivalence:

$$[p \rightarrow (q \rightarrow r)] \Leftrightarrow [(p \wedge \neg r) \rightarrow \neg q]$$

p	q	r	$q \rightarrow r$	$p \rightarrow (q \rightarrow r)$	$\neg r$	$p \wedge \neg r$	$\neg q$	$(p \wedge \neg r) \rightarrow \neg q$
0	0	0	1	1	1	0	1	1
0	0	1	1	1	0	0	1	1
0	1	0	0	1	1	0	0	1
0	1	1	1	1	0	0	0	1
1	0	0	1	1	1	1	1	1
1	0	1	1	1	0	0	1	1
1	1	0	0	0	1	1	0	0
1	1	1	1	1	0	0	0	1

5 (15 pts) Given $U = \{ r, s, t, u, v, w, x, y, z \}$, $A = \{ r, s, v, x, z \}$,

$B = \{ r, t, u, v, z \}$ and $C = \{ s, t \}$

a- Find $(A \oplus B) \cap (B - C)$

b- Find the power set of $C - B$

c- Find $(A - B) \times (\overline{B} \cap C)$

d- Find the cardinal of $\overline{C} - B$

6 (16 pts) Given the relationship \mathcal{R} on set $\{0,1,2,3,4\}$ such that

$\mathcal{R} = \{(a,b); a + b = 2k \text{ where } k \in \mathbb{Z}\}$

a) List the elements of \mathcal{R}

b) List the properties of \mathcal{R}

c) Represent the relation using a digraph.

d) Is the relation an equivalence relation? Justify your answer if yes list the equivalence classes.

7 (11 pts) A group of 191 students, of which 10 are taking French, business and music; 36 are taking French and Business; 20 are taking French and music, 18 are taking business and music; 65 are taking French; 76 are taking business and 63 are taking music.

a- How many are taking French but not business?

b- How many are taking business and neither French nor music?

c- How many are taking French or Business (or both)?

d- How many are taking music or French (or both) but not business?

e- How many are taking none of the three subjects?